Absolutely convergent: $\sum a_n$ is ABS conv if $\sum |a_n|$ is convergent. eg. $\sum \frac{(-1)^n}{n^2}$

Conditionally convergent: If $\sum a_n$ is NOT ABS conv but convergent, then $\sum a_n$ is conditionally conv. eg. $\sum \frac{(-1)^n}{n}$

Series

- **nth term test for divergence:** If $\lim_{n\to\infty} a_n$ does not exist or if $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

- **The p-series:** $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if $p > 1$ and divergent if $p \le 1$. a: first term; r: common ratio.

- **Geometric:** If $|r| < 1$ then $\boxed{\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}}$

- **The Integral Test:** Suppose f is a continuous, positive, decreasing function on $[1, \infty)$ and let $a_n = f(n)$. Then

  Good for $\sum \frac{1}{n(\ln n)^p}$

  $f(n)$ decreasing $\Leftrightarrow f'(n) < 0$

  (i) If $\int_1^{\infty} f(x)\, dx$ is convergent, then $\sum_{n=1}^{\infty} a_n$ is convergent.

  (ii) If $\int_1^{\infty} f(x)\, dx$ is divergent, then $\sum_{n=1}^{\infty} a_n$ is divergent.

- **The Comparison Test:** Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms.

  (i) If $\sum b_n$ is convergent and $a_n \le b_n$ for all n, then $\sum a_n$ is also convergent.

  (ii) If $\sum b_n$ is divergent and $a_n \ge b_n$ for all n, then $\sum a_n$ is also divergent.

- **The Limit Comparison Test:** Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms. If

  Good for Ratio of two Polynomials

  $$\lim_{n\to\infty} \frac{a_n}{b_n} = c$$

  where c is a finite number and $c > 0$, then either both series converge or both diverge.

  e.g. $\sum \frac{\sqrt{n+1}}{4n^2-3}$

Power Series: $\sum c_n(x-a)^n$. Apply Ratio Test to $a_n = c_n(x-a)^n$ to find Radius of Conv as $|x-a| < R$.

Representation: $\frac{1}{1-▨} = \sum_{n=0}^{\infty} ▨^n$, $|▨| < 1$.

- **Alternating Series Test:** If the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$ satisfies eg. $\sum \frac{(-1)^{n+1}}{n}$

  (i) $0 < b_{n+1} \le b_n$ for all n (b_n decreasing)

  (ii) $\lim_{n\to\infty} b_n = 0$

  then the series is convergent.

- **The Ratio Test** Good for product/ratio of power functions and exponential function

  (i) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent. eg. $\sum \frac{n^2}{3^n}$

  (ii) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L > 1$ or $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \infty$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

  (iii) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$, the Ratio Test is inconclusive.

- Taylor Series: $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$

- **Maclaurin Series**: $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$

- **Taylor's Inequality** If $|f^{(n+1)}(x)| \le M$ for $|x-a| \le d$, then the remainder $R_n(x)$ of the Taylor series satisfies the inequality

  $$|R_n(x)| \le \frac{M}{(n+1)!}|x-a|^{n+1} \quad \text{for } |x-a| \le d$$

- Some Power Series (Maclaurin Series)

  - $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots$ $R = \infty$
  - $\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$ $R = \infty$
  - $\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$ $R = \infty$
  - $\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$ $R = 1$
  - $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \cdots$ $R = 1$

Integrals

- **Volume**: Suppose $A(x)$ is the cross-sectional area of the solid S perpendicular to the x-axis, then the volume of S is given by

$$V = \int_a^b A(x)\ dx$$

- **Work**: Suppose $f(x)$ is a force function. The work in moving an object from a to b is given by:

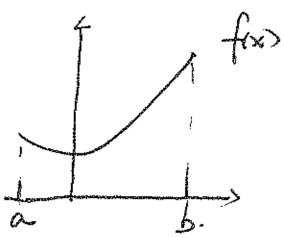
$$W = \int_a^b f(x)\ dx$$

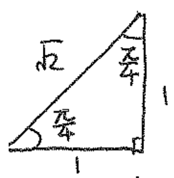
- $\int x^n dx = \frac{1}{n+1} \cdot x^{n+1} + C, \quad n \neq -1.$
- $\int \frac{1}{x}\ dx = \ln|x| + C$
- $\int \tan x\ dx = \ln|\sec x| + C$
- $\int \sec x\ dx = \ln|\sec x + \tan x| + C$
- $\int a^x\ dx = \frac{a^x}{\ln a} + C \qquad \text{for } a \neq 1$
- **Integration by Parts**:

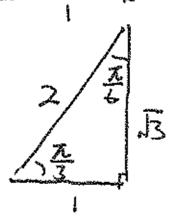
$$\int u\ dv = uv - \int v\ du$$

- Arc Length Formula:

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\ dx$$







Derivatives

- $\frac{d}{dx}(\sinh x) = \cosh x \qquad \frac{d}{dx}(\cosh x) = \sinh x$
- Inverse Trigonometric Functions:

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\csc^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$$

- If f is a one-to-one differentiable function with inverse function f^{-1} and $f'(f^{-1}(a)) \neq 0$, then the inverse function is differentiable at a and

$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$$

Hyperbolic and Trig Identities

- Hyperbolic Functions

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \qquad \operatorname{csch}(x) = \frac{1}{\sinh x}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \operatorname{sech}(x) = \frac{1}{\cosh x}$$

$$\tanh(x) = \frac{\sinh x}{\cosh x} \qquad \coth(x) = \frac{\cosh x}{\sinh x}$$

- $\cosh^2 x - \sinh^2 x = 1$
- $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
- $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
- $\sin(2x) = 2\sin x \cos x$
- $\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
- $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
- $\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$